State Space Design: Pole Placement Examples

MEM 355 Performance Enhancement of Dynamical Systems

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Examples

- In the following examples we will
  - Examine open loop modes
  - Design controller using pole placement
  - Compute equivalent compensator
  - Perform root locus analysis of feedback loop

Example: F-16

landing approach longitudinal dynamics

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.507 & -3.861 & 0 & -32.17 \\ -0.00117 & -0.5164 & 1 & 0 \\ -0.000129 & 1.4168 & -0.4932 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ -0.0717 \\ -1.645 \\ 0 \end{bmatrix} \delta_E$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix}$$

$$\text{phugoid: } \lambda = -0.0438167 \pm j0.206461 \quad h = \begin{bmatrix} 0.999978 \\ 0.000484 \\ 0.001343 \\ -0.000272 \end{bmatrix} \pm j \begin{bmatrix} 0 \\ 0.0002676 \\ 0.0002264 \\ -0.0064497 \end{bmatrix}$$

$$\text{short period: } \lambda = -1.7036,\ 0.730937 \quad h = \begin{bmatrix} -0.994287 \\ -0.063373 \\ 0.074073 \\ -0.043481 \end{bmatrix}, \begin{bmatrix} 0.999508 \\ -0.014171 \\ -0.016507 \\ -0.022584 \end{bmatrix}$$

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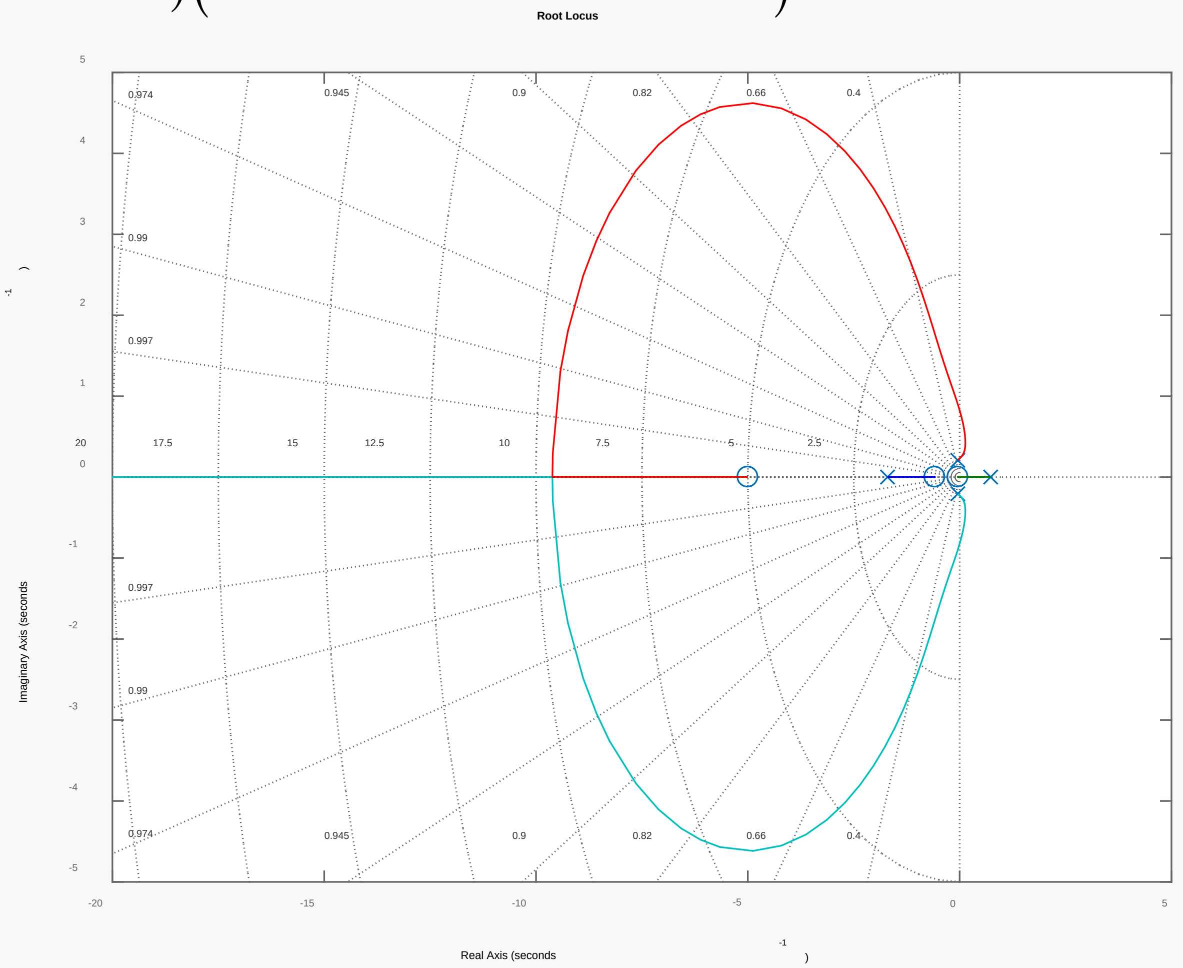
F-16: PI Control

$$G_p(s) = 1.645 \frac{s(s+0.0423101)(s+0.586543)}{(s-0.730937)(s+1.7036)(s^2+0.0876334s+0.044546)}$$

$$G_c(s) = \frac{s+5}{s}$$



Example: F-16 state feedback

Desired poles -

short period: $\lambda_{1,2} = -1.25 \pm j2.16506 \quad (\omega = 2.5, \rho = 0.5)$

phugoid: $\lambda_{3,4} = -0.01 \pm j0.0994987 \quad (\omega = 0.1, \rho = 0.1)$

$$K = \begin{bmatrix} 0.004076 & 3.87578 & 0.718424 & 0.095189 \end{bmatrix}$$

Design via pole placement – requires an observer

Example: F-16 Rynaski “robust observer”

"place observer poles at LHP plant zeros, remainder are placed arbitrarily"

$$\lambda = 0, -0.04231, -0.5865, -1$$

$$L^T = \begin{bmatrix} 0.168343 & -1.02106 & -0.56851 & -1 \end{bmatrix}$$

$$G_p(s) = 1.645 \frac{s(s+0.0423101)(s+0.586543)}{(s-0.730937)(s+1.7036)(s^2+0.0876334s+0.044546)}$$

$$G_c(s) = 4.46035 \frac{(s+1.234)(s^2+0.02967s+0.02198)}{s(s+0.04231)(s+0.5866)(s+2.46)}$$

Equivalent compensator

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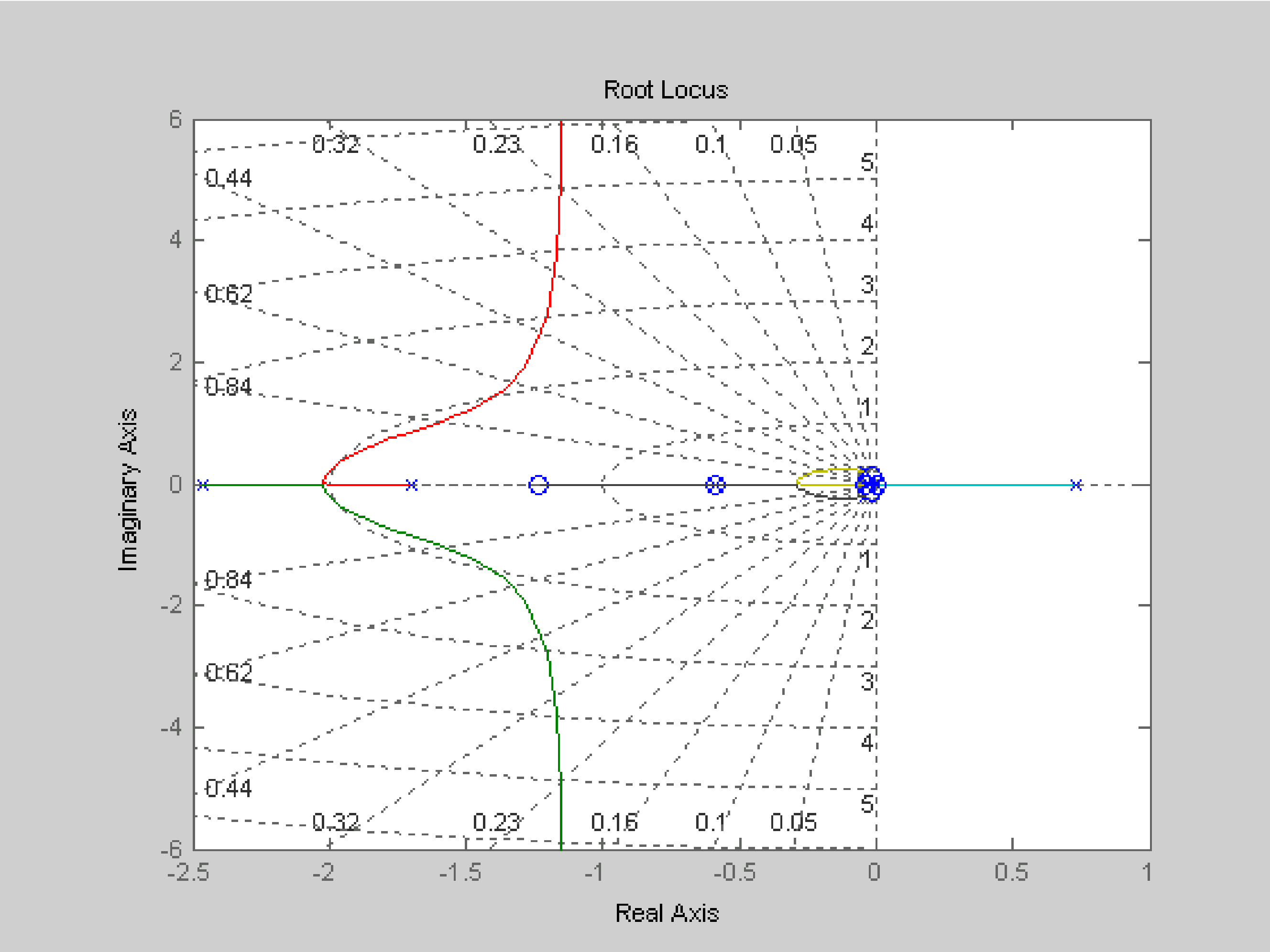
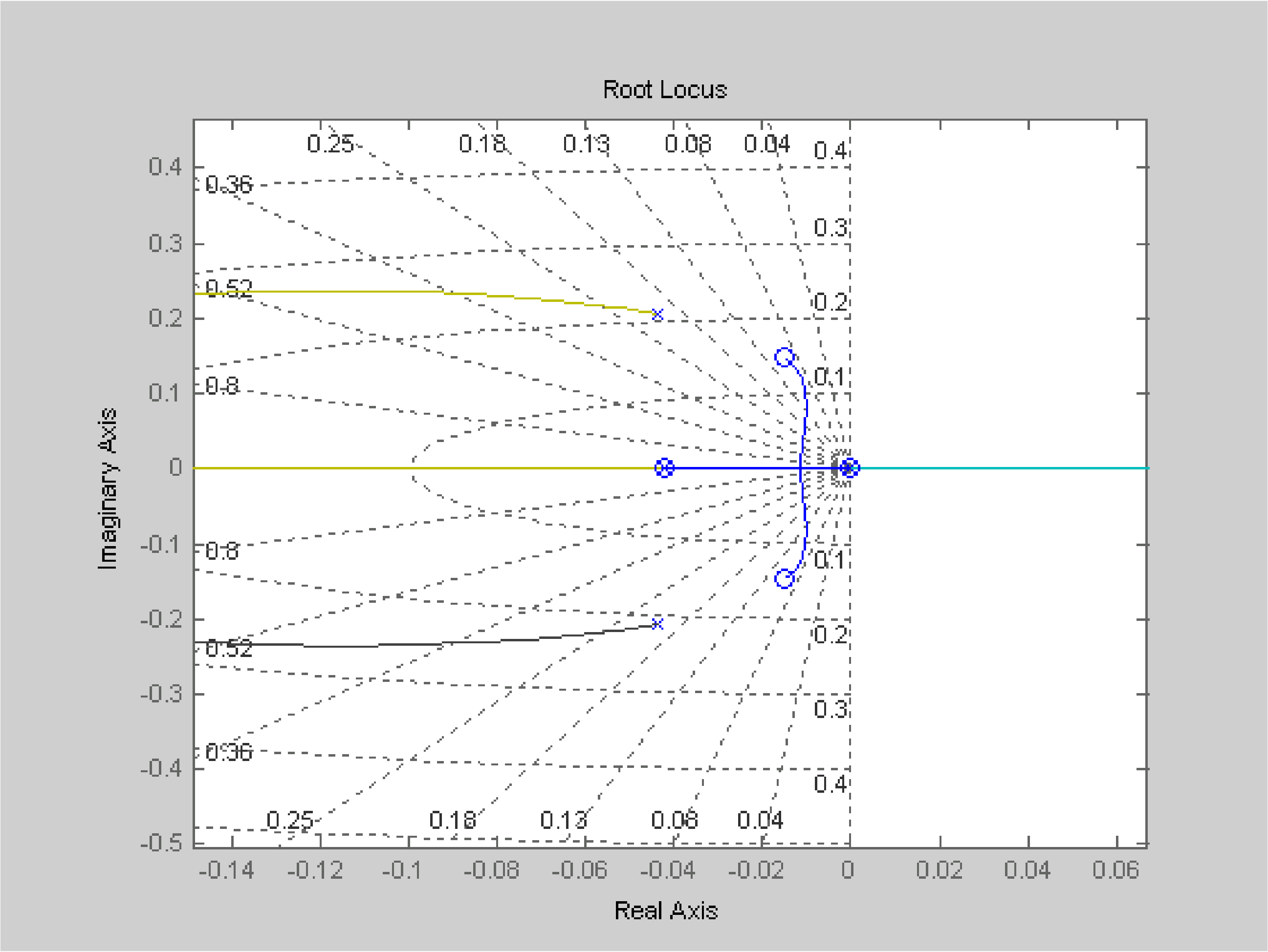
Root Locus

$$G_p(s) = 1.645\frac{s(s+0.0423101)(s+0.586543)}{(s-0.730937)(s+1.7036)(s^2+0.0876334s+0.044546)}$$

$$G_c(s) = 4.46035\frac{(s+1.234)(s^2+0.02967s+0.02198)}{s(s+0.04231)(s+0.5866)(s+2.46)}$$

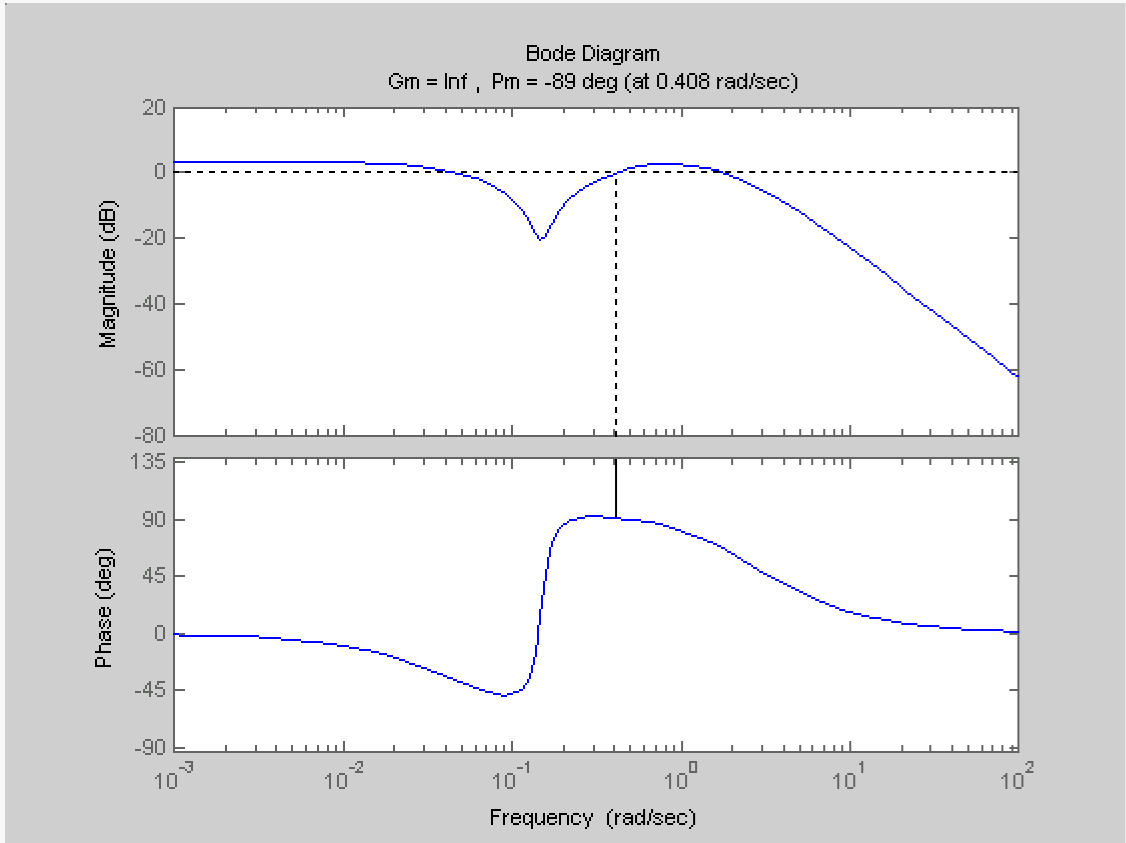


Root Locus 2

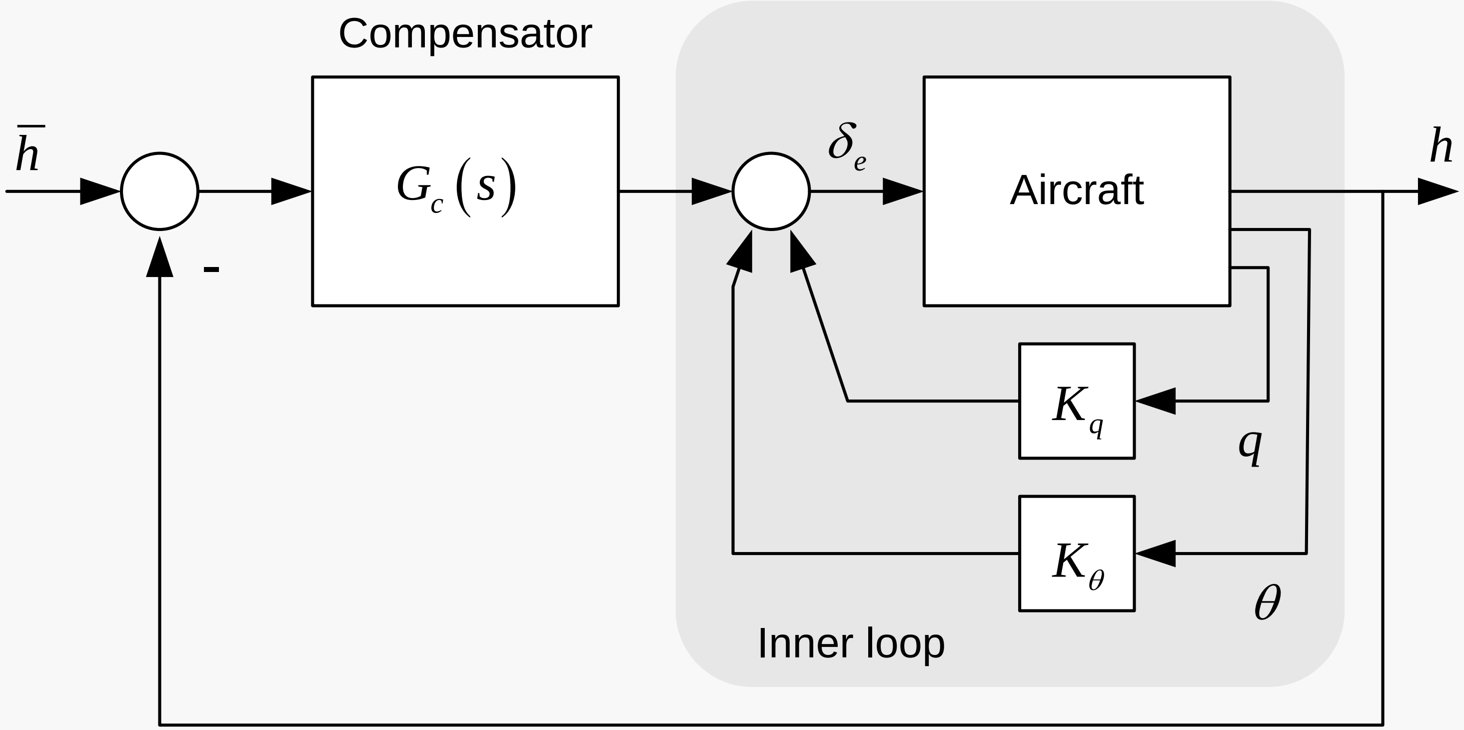


Root Locus
Imaginary Axis
Real Axis
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.14
-0.12
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.25
0.18
0.13
0.08
0.04
0.36
0.52
0.8

Margins


Bode Diagram
Gm = Inf , Pm = -89 deg (at 0.408 rad/sec)
Magnitude (dB)
20
0
-20
-40
-60
-80
Phase (deg)
135
90
45
0
-45
-90
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

Boeing 747-400
altitude hold controller



Boeing 747 Dynamics (cruise)

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} -0.006 & 0.0263 & 0 & -32.2 & 0 \\ -0.0941 & -0.624 & 820 & 0 & 0 \\ -0.000222 & -0.00153 & -0.668 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 830 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} + \begin{bmatrix} 0 \\ -32.7 \\ -2.08 \\ 0 \\ 0 \end{bmatrix} \delta_e$$

$$h = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix}$$

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Boeing 747 Open Loop Longitudinal Modes-1

```
>> A=[-0.0064 0.0263 0 -32.2 0;-0.0941 -0.624 820 0 0;
        -.000222 -0.00153 -0.668 0 0;0 0 1 0 0;0 -1 0 830 0];
>> eig(A)
ans =
   0.0000 + 0.0000i
  -0.6463 + 1.1211i
  -0.6463 - 1.1211i
  -0.0029 + 0.0098i
  -0.0029 - 0.0098i
```

Vertical translation

Short period

Phugoid

Boeing 747 Open Loop Longitudinal Modes-2

```
>> [V,D]=eig(A)

V =
```

0.0000	-0.0116 + 0.0037i	-0.0116 - 0.0037i	-0.0358 - 0.0176i	-0.0358 + 0.0176i
0.0000	-0.9368 + 0.0000i	-0.9368 + 0.0000i	0.0053 + 0.0026i	0.0053 - 0.0026i
0.0000	0.0000 - 0.0013i	0.0000 + 0.0013i	-0.0000 - 0.0000i	-0.0000 + 0.0000i
0.0000	-0.0009 + 0.0005i	-0.0009 - 0.0005i	0.0000 + 0.0000i	0.0000 - 0.0000i
1.0000	0.1816 - 0.2988i	0.1816 + 0.2988i	0.9992 + 0.0000i	0.9992 + 0.0000i

Short period

Vertical translation

Phugoid

Boeing 747 Inner Loop Design

```
A=[-0.0064 0.0263 0 -32.2 0;-0.0941 -0.624 820 0 0;-
.000222 -0.00153 -0.668 0 0;0 0 1 0 0;0 -1 0 830 0];
B=[0;-32.7;-2.08;0;0];
C=[0 0 0 0 1];
poles=[0,-2.25+2.99i,-2.25-2.99i,-0.0105,-0.0531];
Kinner=place(A,B,poles)
Kinner =
   -0.0008   -0.0054   -1.4845   -0.6517         0
eig(A-B*Kinner)
ans =
        0
  -2.2500 + 2.9900i
  -2.2500 - 2.9900i
  -0.0531
  -0.0105
```

K_q K_θ

Small contribution, so we'll drop these two terms. Thus, the implementation does not need an observer.

```
[0,-0.6463+1.1211i,-0.6463-1.1211i,-0.0029+0.0098i,-0.0029-0.0098i]
```

original poles

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Boeing 747 cont'd

RHP zero

$$\delta_e \to h: G(s) = \frac{32.7(s+0.0045)(s+0.5645)(s-5.61)}{s(s+0.003 \pm j0.0098)(s+0.6463 \pm j1.1211)}$$

phugoid short-period

Choose: $K_q = -1.4845,\ K_\theta = -0.6517$

Inner loop improves stability

$$A \to A_p = A + b\begin{bmatrix} 0 & 0 & -1.4845 & -0.6517 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0064 & 0.0263 & 0 & -32.2 & 0 \\ -0.0941 & -0.624 & 721 & -21 & 0 \\ -0.0002 & -0.0015 & -3.76 & -1.36 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 830 & 0 \end{bmatrix}$$

New *A* matrix

Note zeros are unchanged

$$G \to G_p(s) = \frac{32.7(s+0.0045)(s+0.5645)(s-5.61)}{s(s+2.25 \pm j2.99)(s+0.0105)(s+0.0531)}$$

Outer Loop Design Computations-1

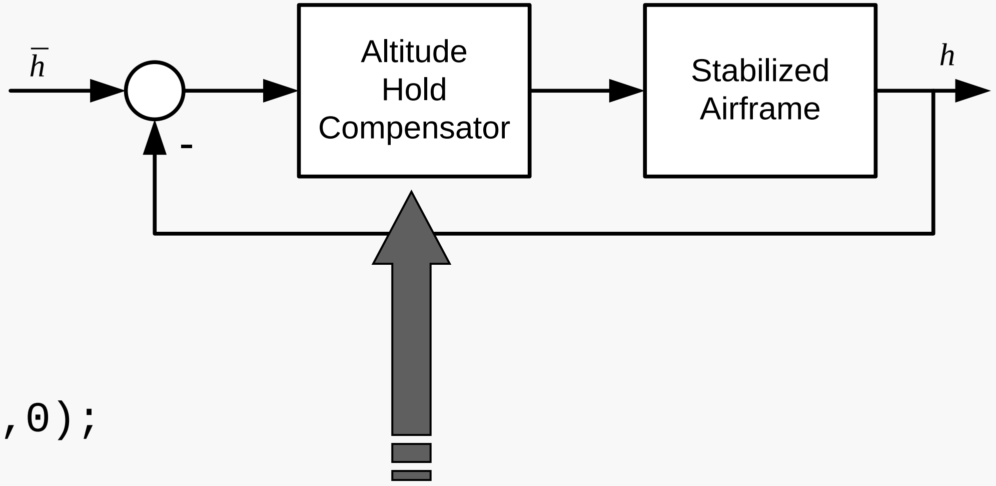
```
>> A=[-0.0064 0.0263 0 -32.2 0;-0.0941 -0.624 761 -196.2 0;
                  -.0002 -0.0015 -4.41 -12.48 0;0 0 1 0 0;0 -1 0 830 0]
A =
   -0.0064    0.0263         0  -32.2000         0
   -0.0941   -0.6240  761.0000 -196.2000         0
   -0.0002   -0.0015   -4.4100  -12.4800         0
         0         0    1.0000         0         0
         0   -1.0000         0  830.0000         0
>> b=[0;-32.7;-2.08;0;0]
b =
         0
  -32.7000
   -2.0800
         0
         0
>> p=[-.0045;-.145;-.513;-2.25+i*2.98;-2.25-i*2.98];
```

Computations 2

```
>> K=place(A,b,p)
K =
   -0.0011    0.0016   -0.0843   -1.6011   -0.0010
>> eig(A-b*K)
ans =
  -2.2500 + 2.9800i
  -2.2500 - 2.9800i
  -0.0045
  -0.5130
  -0.1450
>> c=[0,0,0,0,1];
>> poles=[-0.0045,-5.645,-9,-10,-11];
>> L=place(A',c',poles)'
L =
   -6.5323
  915.2339
   -2.7283
    1.4615
   30.6091
```

Computations 3

```
>> eig(A-L*c)
ans =
   -0.0045
  -11.0000
  -10.0000
   -9.0000
   -5.6450
>> Ac=A-b*K-L*c;
>> Bc=L;
>> Cc=K;
>> Gcss=ss(Ac,Bc,Cc,0);
>> Gc=tf(Gcss);
>> zpk(Gc)
Zero/pole/gain:
-0.64364 (s+0.5803) (s+0.004708) (s^2  + 4.517s + 14.29)
--------------------------------------------------------
 (s+13.22) (s+5.644) (s+0.004507) (s^2  + 16.9s + 79.2)
```

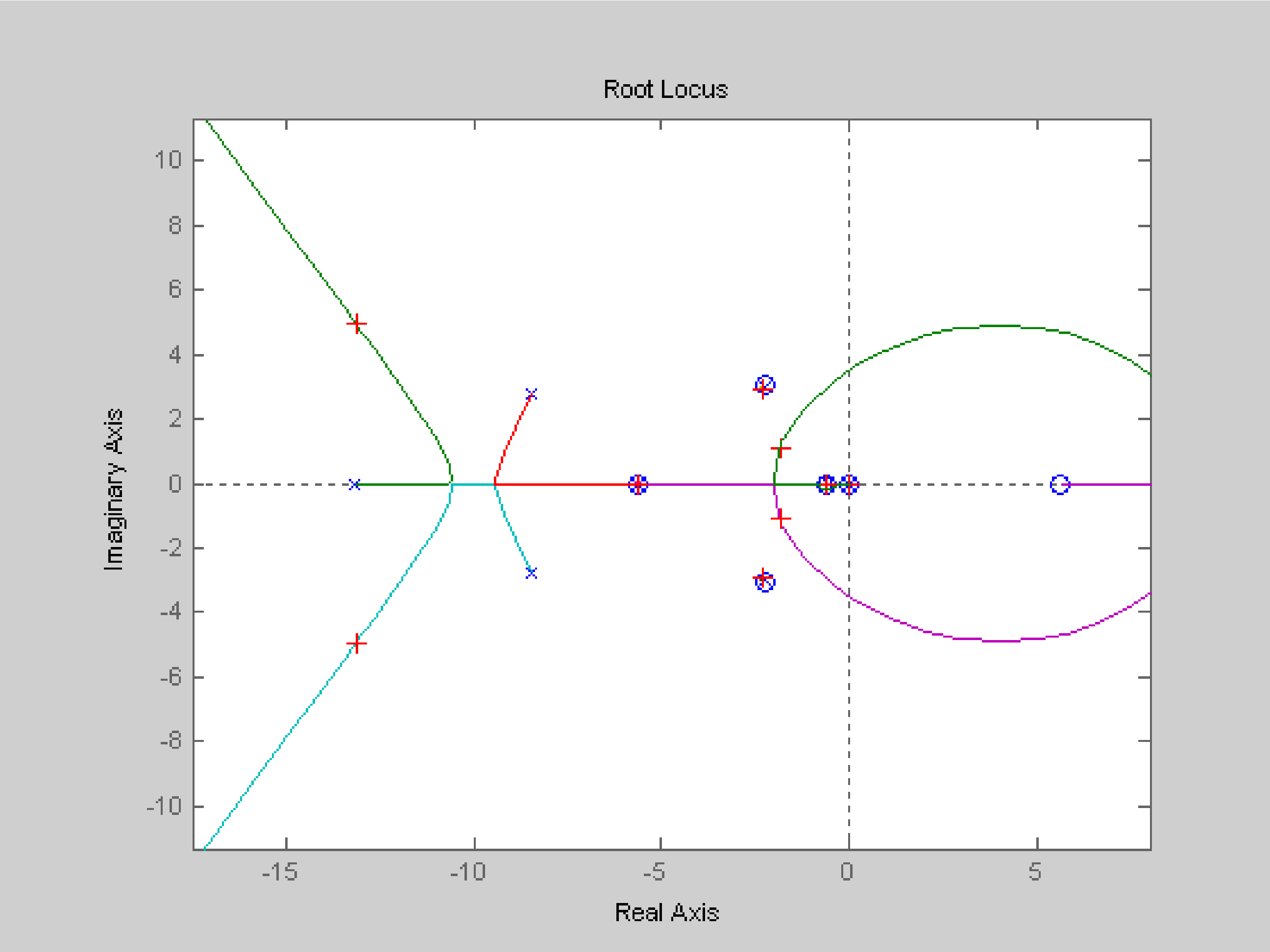


Computations Summary

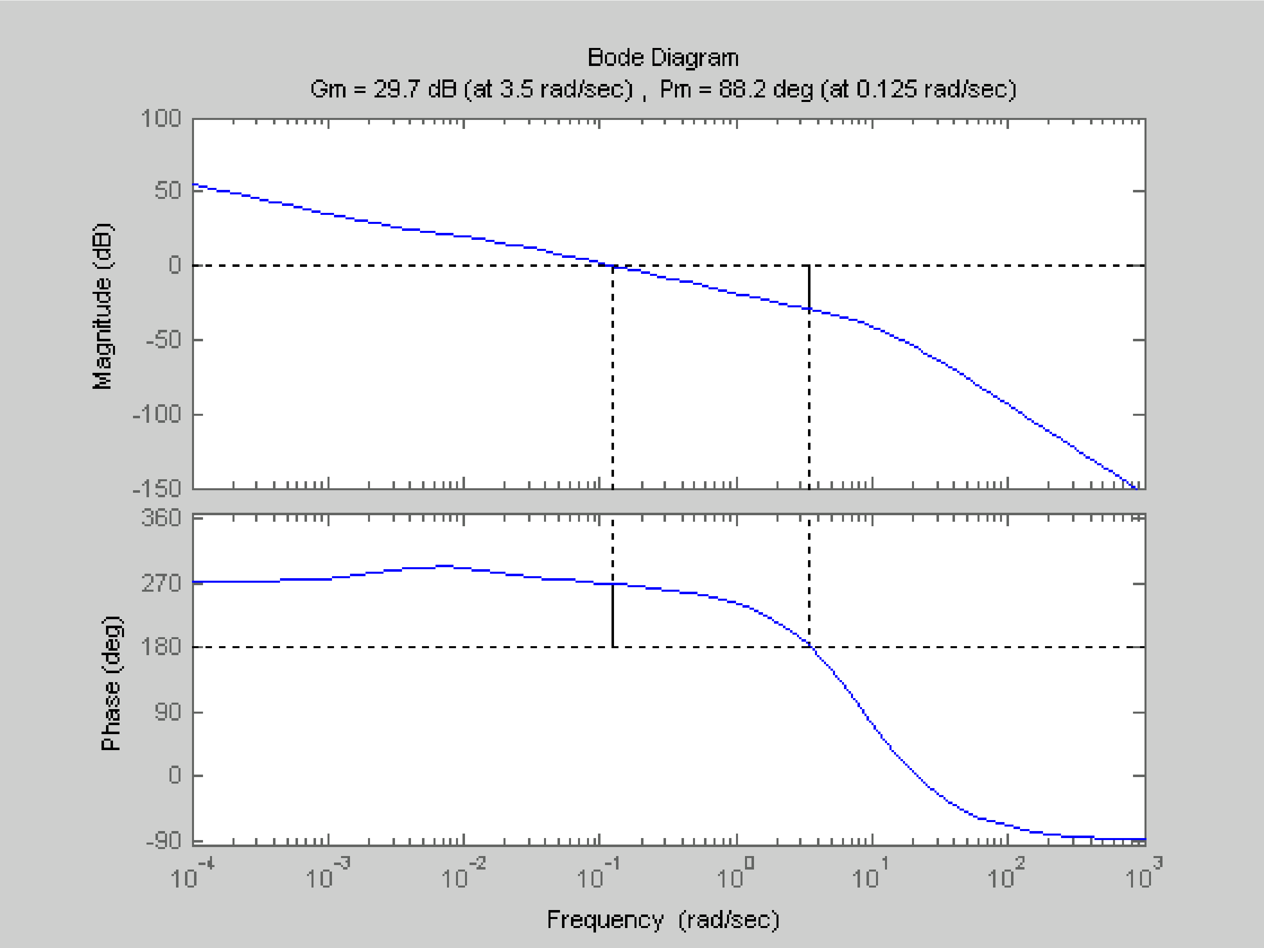
$$G_p(s) = \frac{32.7(s + 0.0045)(s + 0.5645)(s - 5.61)}{s(s + 2.25 \pm j2.99)(s + 0.0105)(s + 0.0531)}$$

$$G_c(s) = \frac{-0.644(s + 0.5803)(s + 0.004708)(s + 2.258 \pm j3.0314)}{(s + 13.22)(s + 5.644)(s + 0.0045)(s + 8.45 \pm j2.7924)}$$

Root Locus


Root Locus
Imaginary Axis
Real Axis
10
8
6
4
2
0
-2
-4
-6
-8
-10
-15
-10
-5
0
5

Margins



Bode Diagram
Gm = 29.7 dB (at 3.5 rad/sec) , Pm = 88.2 deg (at 0.125 rad/sec)
Magnitude (dB)
100
50
0
-50
-100
-150
Phase (deg)
360
270
180
90
0
-90
10^-4
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/sec)